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SOFT COMPUTING IN DECISION MAKING AND IN MODELING IN ECONOMICS

Interval-valued intuitionistic fuzzy multi-attribute second-order decision making based on partial connection numbers of set pair analysis

Qing Shen[1,3] · Xiongtao Zhang[2,3] · Jungang Lou[2,3] · Yong Liu[4] · Yunliang Jiang[2,3]

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Abstract

Multi-attribute decision making (MADM) with attribute values as interval-valued intuitionistic fuzzy numbers (IVIFNs) is essentially a second-order decision making problem with uncertainty. To this end, the partial connection number (PCN) of set pair analysis is applied to MADM with IVIFNs. The PCN is an adjoin function of the connection number (CN), and its calculation process reflects the contradictory movement of the connection component in the CN at various micro-levels. It is the main mathematical tool of multi-level analysis method for the macro state and micro-trend. First, we convert IVIFNs into ternary connection numbers (TCNs); then, we calculate the first-order and second-order total PCNs for TCNs. According to the uncertainty analysis of the first-order total PCN, the possible ranking (first-order ranking) of the schemes in the uncertain environment is given, and the deterministic ordering (second-order ranking) of the schemes is given according to the value of the second-order total PCN to meet the needs of different decision making levels. The practical application shows that the method presented is novel, and the results are in line with the uncertainty decision making. Furthermore, the current status and development trend of schemes are taken into account to make decision making progress more reasonable and operable.

Keywords Interval-valued intuitionistic fuzzy numbers · Multi-attribute second- order decision making · Partial connection number · Set pair analysis

1 Introduction

Since Atanassov (1986) proposed the concept of intuitionistic fuzzy sets (IFSs), scholars have introduced different high-order fuzzy sets (Atanassov and Gargov 1989; Zhao and Xiao 2012)for solving uncertainty problems. Among them, the interval-valued intuitionistic fuzzy set (IVIFS) expresses the membership degree, nonmembership degree and unknown degree as closed subintervals of [0,1], thus expanding the ability of the IFSs to process uncertain information. In recent years, research on interval-valued intuitionistic fuzzy multi-attribute decision making (IVIFMADM) has attracted wide attention (Kumar and Chen 2021a; Jia and Liu 2021; Peng et al. 2021; Shen et al. 2020; Gupta et al. 2018; Chen and Huang 2017; Nayagam et al. 2017). Chen et al. (2012) proposed a new intuitionistic fuzzy weighted average operator and interval-valued intuitionistic fuzzy weighted average operator and applied them to the MADM based on interval-valued intuitionistic fuzzy values (IVIFVs). Kumar and Chen (2021a) proposed a new MADM method based on converted

✉ Yunliang Jiang
jyl@zjhu.edu.cn

Qing Shen
sq@zjhu.edu.cn

Xiongtao Zhang
02032@zjhu.edu.cn

Jungang Lou
ljg@zjhu.edu.cn

Yong Liu
yongliu@iipc.zju.edu.cn

1 School of Science and Engineering, Huzhou College, Huzhou 313000, China

2 School of Information Engineering, Huzhou University, Huzhou 313000, China

3 Zhejiang Province Key Laboratory of Smart Management and Application of Modern Agricultural Resources, Huzhou University, Huzhou 313000, China

4 Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China

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decision matrices, probability density functions and IVIFVs. Nguyen (2016) proposed a distance-based knowledge measure and constructed an information interval-valued entropy measure for IVIFS. Garg (2016) presented the generalized improved score function for ranking the IVIFS. Wang and Wan (2020) investigated the order relation of IVIFVs based on possibility degree and divergence degree and proposed a method to solve multi-attribute group decision making (MAGDM) problems with IVIFVs.

How to effectively deal with uncertain information in a decision making process is the key to solving MADM problems. The traditional method is to convert uncertain information into deterministic information; however, in the process of decision making transformation or fuzzy information aggregation, uncertain information will be lost. Therefore, the credibility of decision making results is not high. Additionally, it is difficult to further analyze and compare the decision making results that do not meet the requirements of flexible decision making. Zhao (1989) proposed a new system analysis method, the set pair analysis (SPA) theory. Set pair is a basic unit consisting of two sets. Why should such a basic unit be given? First, it comes from thinking about the paradox in set theory, especially Russells paradox; second, it is inspired by the uncertainty principle of physics; third, it is enlightened by the philosophical paired principle proposed by the idea that objective things are in universal connection and the unity of opposites is the universal law of the objective world. On the other hand, we get the knowledge from the practical research. Faced with a concrete objective object O, there is not only a certain set A of the object, but also an uncertain set B about the object; and the uncertain set B is always associated with the uncertain set A. It is only when the two sets are combined to describe the given objective object O, and a comprehensive objective description of the object O can be obtained and vice versa. A certain set A describing the given object O is always associated with an uncertain set B about the object. In other words, when trying to determine that an element in a collection is the entirety of an object, there must be some elements that are uncertain about whether they belong to the set, such as the hairdresser in the barbers paradox (global uncertainty). If you want to exclude this hairdresser, your research object becomes a kind of partial thing, and the conclusions obtained will make the mistake of partiality (partial uncertainty). In short, for a given objective object, it is necessary to use two sets at the same time to describe the things people want to study.

Generally speaking, the two sets must be a certain set and an uncertain set. In special cases, either two sets are uncertain sets, or both are certain sets. Since such two sets describe the same objective object in a complementary way, they should naturally be placed in the same research unit to reflect the interrelationship between the two sets in their original sense, and this unit should be vividly called set pair. The mathematical tool for describing the deterministic measure and uncertain measure of two sets and their relationship in set pairs is the connection number (CN) of SPA. The advantage of SPA lies in the ability to simultaneously consider the certainty and uncertainty of decision information as a system. Furthermore, it recognizes that issues such as such as fuzziness, randomness and uncertainty need to be comprehensively addressed in decision making process. SPA has attracted extensive attention from academic circles because of its advantages in solving uncertain information decision making, and it has been successfully applied in MADM (Cao et al. 2018; Garg and Kumar 2018a, b, c, 2020, b; KumarK 2018; Kumar and Garg 2018). Cao et al. (2018) overcame the shortcomings of the existing methods, by using the CN to represent the interval-valued fuzzy information and ranking the schemes based on the set pair potential and fully considered the advantages of SPA. Kumar and Chen (2021b) proposed a new MADM method based on INIFVs, the proposed score function of CNs and the SPA theory, where IVIFVs are converted into CNs and the optimal weights of the attributes are calculated from interval-valued intuitionistic fuzzy weights of the attributes. Shen et al. (2021) presented an innovative decision framework with the expectation + range binary CN and first applied to hesitant fuzzy MADM. SPA is preferred because CN has the dual characteristics of certainty and uncertainty, which is isomorphic to the uncertainty of the interval number with upper and lower bounds and arbitrary values in its range.

In this paper, it is considered that an obvious deficiency in the study on IVIFMADM in the above literature is that it does not recognized that IVIFMADM is a multi-order uncertainty decision making. It is illogical and inconsistent with IVIFMADM to give a unique ranking of solutions in a global sense without order. The interval number itself is a real number that can be legally taken as an infinite number of different values in a given interval, which results in a specific IVIFMADM with infinite different rankings for solutions and different optimal solutions. Therefore, it is necessary to study the corresponding conditions and the order of the optimal solution to better meet the actual decision making needs. This paper presents a new algorithm for IVIFMADM based on partial connection numbers (PCNs) is presented, which solves this problem to a certain extent. PCN is a kind of adjoin function of CN Zhao (2005), and it is also the main mathematical tool of system state trend analysis method based on SPA. We utilize PCN to propose a second-order decision making method, which can take into account the current status evaluation and development trend evaluation for decision making schemes. To do so, the remainder of this paper is set out as follows. Section 2 introduces some concepts related to SPA and PCN. In Sect. 3, IVIFMADM is described and different decision making procedures are developed in three cases. In Sect. 4, three examples for IVIFMADM are given to demonstrate the applications and effectiveness of the proposed methods. Finally, Sect. 5 gives some concrete conclusions.

2 Introduction of PCN

PCN is an adjoint function of CN, and CN is the characteristic function of set pair. In this paper, set pair analysis and its CN are briefly introduced.

2.1 SPA and CN

SPA is a systematic mathematical theory for dealing with uncertainty; it was proposed by Zhao Keqin, a Chinese scholar, in 1989. Set pair refers to a research unit consisting of two sets with a certain relationship. In IVIFMADM, the decision making conditions and results, the uncertainty factors and certainty factors, and the decision making algorithms and schemes sequencing can be formalized into set pairs under certain conditions.

Let set pairs consisting of the set E and set F be represented by $H = (E, F)$; let A represent the certain relation number of E and F sets in a given universe, and let B represent the uncertain relation number in a given universe. Then, the characteristic functions of the set pairs $H = (E, F)$ are defined as follows.

Definition 2.1 Let A, B be nonnegative real numbers, $i \in [-1, 1]$, and let u indicate the CN, then

$$u = A + Bi \tag{1}$$

is called a binary connection number (BCN). In Eq. (1), let $N = A + B$, $\mu = u/N$, $a = A/N$, $b = B/N$, and the following formula is derived from Eq. (1)

$$\mu = a + bi \tag{2}$$

where $a+b = 1, a \in [0, 1], b \in [0, 1], i \in [-1, 1]$. Equation (2) is sometimes called the binary connection degree, connection degree or CN for short. $A(a)$ and $B(b)$ are referred to as connection components of the CN, where $A(a)$ is the certainty measure connection component of the CN, and $B(b)$is the uncertainty measure connection component.

Definition 2.2 Let $\mu = a + bi$ be a BCN, the combination of the certainty measure a and the uncertainty measure bi when $i = 1$ is

$$r = \sqrt{a^2 + b^2} \tag{3}$$

which is called as the modulus of the BCN.

Definition 2.3 Let A, B, C be nonnegative real numbers, $j = -1$, $i \in [-1, 1]$, then

$$U = A + Bi + Cj \tag{4}$$

is a ternary connection number (TCN). Let $A + B + C = N$, $\mu = \frac{U}{N}$, $a = \frac{A}{N}$, $b = \frac{B}{N}$, $c = \frac{C}{N}$, then

$$\mu = a + bi + cj \tag{5}$$

where $a + b + c = 1$, $a \in [0, 1]$, $b \in [0, 1]$, $c \in [0, 1]$, $i \in [-1, 1]$, $j = -1$. Equation (5) is sometimes referred to as the ternary connection degree.

Equations (4) or (5) also referred to as the identity-discrepancy-contrary connection number (identity-discrepancy-contrary connection degree), the meaning of which can be explained by the connection component, wherein $A(a)$ is called the identity degree, $B(b)$ is called the discrepancy degree and $C(c)$ is called the contrary degree. The items $A(a)$, $B(b)$ and $C(c)$ in the connection number (degree) are called the connection component of the CN $U(\mu)$, where $A(a)$ is called the identical component, $B(b)$ is called the discrepant component, and $C(c)$ is called the contradictory component, which are collectively called the connection component. i and j are called the value coefficients of the connection components $B(b)$ and $C(c)$, respectively.

From the TCN, we can derive multi-CNs, such as the quaternion CN and quintuple CN. In this paper, only the BCN and TCN are used.

2.2 PCN of the TCN

The PCN is an adjoint function defined based on the assumption that relative motion exists in the connection component of CN. The PCN of the TCN $\mu = a+bi+cj$ includes the first-order partial positive CN (first-order positive motion), the first-order partial negative CN (first-order negative motion), the first-order total PCN (the synthetic motion of the first-order positive motion and first-order negative motion), the second-order partial positive CN (second-order positive motion), the second-order negative CN (second-order negative motion), and the second-order total PCN (the synthetic motion of the second-order positive motion and the second-order negative motion). Specific definitions are as follows.

Definition 2.4 Let there be a TCN $\mu = a + bi + cj$, where $a \in [0, 1], b \in [0, 1], c \in [0, 1], a + b + c = 1, i \in [-1, 1]$, $j = -1$, its first-order partial positive CN is

$$\partial^+\mu = \frac{a}{a+b} + \frac{b}{b+c} i^+ \tag{6}$$

and its first-order partial negative CN is

$$\partial^{-}\mu = \frac{b}{a+b}i^{-} + \frac{c}{b+c}j \tag{7}$$

The first-order total PCN is the algebraic sum of the first-order partial positive CN and the first-order partial negative CN, denoted as $\partial^{\pm}\mu$

$$\partial^{\pm}\mu = \partial^{+}\mu + \partial^{-}\mu = \frac{a}{a+b} + \frac{b}{b+c}i^{+} + \frac{b}{a+b}i^{-} + \frac{c}{b+c}j \tag{8}$$

Generally, the PCN of a TCN is defined as the total PCN, Eq. (8) can be abbreviated as

$$\partial^{\pm}\mu = \partial^{+}\mu + \partial^{-}\mu = \frac{a+bi^{-}}{a+b} + \frac{bi^{+}+cj}{b+c} \tag{9}$$

In Eqs. (8) and (9), there are i and j. When calculating, we will encounter the problem of identifying the value i. To eliminate i, we establish the following definition. According to Definition 2.3, let there be a TCN $\mu = a + bi + cj$, where $a \in [o, 1], b \in [o, 1], c \in [o, 1], a + b + c = 1, i \in [-1, 1]$, $j = -1$, we calculate the partial positive CN of Eq. (6), and obtain the second-order partial positive CN of μ, denoted as $\partial^{2\pm}\mu$,

$$\partial^{2\pm}\mu = \partial\left(\partial^{+}\mu\right) = \frac{\frac{a}{a+b}}{\frac{a}{a+b} + \frac{b}{b+c}} \tag{10}$$

The physical meaning of Eq. (10) is that $\partial^{+}a\left(=\frac{a}{a+b}\right)$ is also at the level of $i\partial^{+}b\left(=\frac{b}{b+c}\right)$ before, and it comes from the positive movement of the $i\partial^{+}b\left(=\frac{b}{b+c}\right)$ lever. Therefore, we can obtain the second- order partial positive change rate $\partial^{2\pm}\mu$ of μ with $\frac{a}{a+b}$ divided by $\left(\frac{a}{a+b} + \frac{b}{b+c}\right)$. Similarly, the second-order partial negative change rate $\partial^{2-}\mu$ of μ is defined as follows.

Definition 2.5 Let there be a TCN μ, then the second-order partial negative connection number is

$$\partial^{2-}\mu = \left(\partial^{-}\left(\partial^{-}\mu\right)\right) = \frac{\frac{c}{b+c}}{\frac{b}{a+b} + \frac{c}{b+c}}j \tag{11}$$

Definition 2.6 Let there be a TCN μ, he second-order partial positive connection number of μ is Eq. (10), and the second-order partial negative connection number of μ is Eq. (11). Then, the second-order total partial connection number $\partial^{2\pm}\mu$ of μ is shown in Eq. (12).

$$\begin{aligned}\partial^{2\pm}\mu &= \partial^{2+}\mu + \partial^{2-}\mu = \frac{\frac{a}{a+b}}{\frac{a}{a+b} + \frac{b}{b+c}} + \frac{\frac{c}{b+c}}{\frac{b}{a+b} + \frac{c}{b+c}}j \\ &= \frac{\frac{a}{a+b}}{\frac{a}{a+b} + \frac{b}{b+c}} - \frac{\frac{c}{b+c}}{\frac{b}{a+b} + \frac{c}{b+c}}\end{aligned} \tag{12}$$

Equation (12) is a real number without uncertain coefficients. The physical meaning is as follows: when $\partial^{2\pm}\mu > 0$, the system of the TCN has a positive trend on the second-order level; when $\partial^{2\pm}\mu < 0$, the system of TCN has a negative trend on the second-order level; and when $\partial^{2\pm}\mu = 0$, the system has a positive and negative critical state on the second-order level.

3 IVIFMADM

3.1 IVIFS

Definition 3.1 Atanassov and Gargov (1989). Let X be a nonempty set; then, $\tilde{A} = \left\{\left\langle x, \tilde{\mu}_{\tilde{A}}(x), \tilde{\nu}_{\tilde{A}}(x)\right\rangle | x \in X\right\}$ is called an IVIFS, where $\tilde{\mu}_{\tilde{A}}(x) \subset [0, 1]$, $\tilde{\nu}_{\tilde{A}}(x) \subset [0, 1]$, and $sup\tilde{\mu}_{\tilde{A}}(x) + sup\tilde{\nu}_{\tilde{A}}(x) \leq 1, \forall x \in X$.

Definition 3.2 The basic component of IVIFS is an ordered interval pair composed of the membership interval and non-membership interval of the element x in X. The real number orderly pair consisting of $\tilde{\mu}_{\tilde{A}}(x)$ and $\tilde{\nu}_{\tilde{A}}(x)$ in the IVIFS is called the IVIFN, denoted as $\left(\tilde{\mu}_{\tilde{A}}(x), (\tilde{\nu}_{\tilde{A}}(x)\right)$. For convenience, IVIFN is generally abbreviated as $([a, b], [c, d])$, where $[a, b] \subset [0, 1]$, $[c, d] \subset [0, 1]$, and $b + d \leq 1$.

3.2 Description of IVIFMADM

Suppose the MADM problems with the IVIFNs as follows. Let S be a set of alternatives, A be a set of attributes and W be a set of attribute weights, where $S = \{S_1, S_2, \ldots, S_m\}$, $A = \{a_1, a_2, \ldots, a_n\}$, $W = \{w_1, w_2, \ldots, w_n\}$, and w_t denotes the weight of attribute a_t and $1 \leq t \leq n$. Assume that the feature information of the alternative S_k $(\forall k = 1, 2, \ldots, m)$ with respect to attribute A_t $(\forall t = 1, 2, \ldots, n)$ is given in the form of IVIFN, and all the given IVIFNs form a decision matrix $D = [\alpha_{kt}]_{m\times n}$, where $\alpha_{kt} = \langle[\mu_{kt}^{L}, \mu_{kt}^{U}], [\nu_{kt}^{L}, \nu_{kt}^{U}]\rangle$, $0 \leq \mu_{kt}^{L} \leq \mu_{kt}^{U} \leq 1, 0 \leq \nu_{kt}^{L} \leq \nu_{kt}^{U} \leq 1, \mu_{kt}^{L} + \nu_{kt}^{U} \leq 1, \forall k = 1, 2, \ldots, m$, and $\forall t = 1, 2, \ldots, n$. Try to decide the optimal scheme, and give the preference order of all the schemes.

3.3 Decision making procedure

In the following, we develop an interval-valued intuitionistic fuzzy multi-attribute second-order decision making approach based on PCN, which involves the following three cases:

Case I: Assume that the attribute weights $w_1, w_2, \ldots, w_n$ are known as real numbers:

$$w = \begin{matrix} a_1 & a_2 & \cdots & a_n \\ [w_1 & w_2 & \cdots & w_n] \end{matrix}$$

where w_t denotes the weight of the attribute a_t, $w_t \in [0, 1]$, $1 \leq t \leq n$, $\sum_{t=1}^{n} w_t = 1$. The proposed multi-attribute second-order decision making method based on PCN is presented as follows:

Step I-1: The interval-valued intuitionistic fuzzy decision matrix $D = [\alpha_{kt}]_{m\times n}$ is transformed into the decision matrix $R = [r_{kt}]_{m\times n} = [a_{kt} + b_{kt} i]_{m\times n}$ in the form of a BCN according to the transformation of Eqs. (13–16).

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j = \frac{\mu_A^L + \mu_A^U + \nu_A^L + \nu_A^U}{4} \tag{13}$$

$$\alpha_{kt} = \frac{\mu_{kt}^L + \mu_{kt}^U + \left(1 - \nu_{kt}^L\right) + \left(1 - \nu_{kt}^U\right)}{4} \tag{14}$$

$$b_{kt} = \sqrt{\frac{\left(\mu_{kt}^L - \bar{x}\right)^2 + \left(\mu_{kt}^U - \bar{x}\right)^2 + \left(\nu_{kt}^L - \bar{x}\right)^2 + \left(\nu_{kt}^U - \bar{x}\right)^2}{3}} \tag{15}$$

That is

$$r_{kt} = a_{kt} + b_{kt} i = \frac{\mu_{kt}^L + \mu_{kt}^U + \left(1 - \nu_{kt}^L\right) + \left(1 - \nu_{kt}^U\right)}{4} + \sqrt{\frac{\left(\mu_{kt}^L - \bar{x}\right)^2 + \left(\mu_{kt}^U - \bar{x}\right)^2 + \left(\nu_{kt}^L - \bar{x}\right)^2 + \left(\nu_{kt}^U - \bar{x}\right)^2}{3}}\, i \tag{16}$$

Step I-2: Calculate the weighted comprehensive CN η_k of each scheme S_k, shown as follows:

$$\eta_k = a_k + b_k i = \sum_{t=1}^{n} w_t a_{kt} + \sum_{t=1}^{n} w_t b_{kt} i \tag{17}$$

Step I-3: Construct a decision model based on the TCN, shown as follows:

$$\eta_k^{'} = a_k + b_k i + c_k j \tag{18}$$

where c_k is the operation when the BCN is converted to the TCN, because $a_k + b_k + c_k = 1$, there exists $c_k = 1 - a_k - b_k$. Equation (18)is also referred to as the decision CN.

Step I-4: Calculate the first-order total PCN of the decision model Eq. (18) with Eq. (8), shown as in Eq. (19).

$$\partial^{\pm}\eta_k^{'} = \partial^{+}\eta_k^{'} + \partial^{-}\eta_k^{'} = \frac{a_k}{a_k + b_k} + \frac{b_k}{b_k + c_k} i^{+} + \frac{b_k}{a_k + b_k} i^{-} + \frac{c_k}{b_k + c_k} j \tag{19}$$

Step I-5: Calculate the value of Eq. (19) when i^{+} and i^{-} take various values and determine the corresponding sorting and optimal scheme of Eq. (19) in the case of first-order uncertainties.

Step I-6: Utilize Eq. (12) to calculate the second-order total PCN $\partial^{2\pm}\eta_k^{'} = \partial^{2+}\eta_k^{'} + \partial^{2-}\eta_k^{'}$ of the decision model Eq. (18), then obtain the second-order ranking and optimal scheme.

Step I-7: Based on the results of Step I-5 and Step I-6, give some suggestions for the decision making.

Case II: Assume that the attribute weights $w_1, w_2, \cdots, w_n$ are known as IVIFNs:

$$w = \begin{matrix} a_1 & a_2 & \cdots & a_n \\ \langle[\varpi_1^L, \varpi_1^U], [\rho_1^L, \rho_1^U]\rangle & \langle[\varpi_2^L, \varpi_2^U], [\rho_2^L, \rho_2^U]\rangle & \cdots & \langle[\varpi_n^L, \varpi_n^U], [\rho_n^L, \rho_n^U]\rangle \end{matrix}$$

We get the most desirable alternative(s) according to the following steps: *Step* II-1: Transform the interval-valued intuitionistic fuzzy decision matrix $D = [\alpha_{kt}]_{m\times n}$ into the BCN decision matrix $R = [r_{kt}]_{m\times n} = [a_{kt} + b_{kt} i]_{m\times n}$ by utilizing Eqs. (13–16).

Step II-2: Transform the interval-valued intuitionistic fuzzy attribute weight w_t into the BCN weight $\tilde{w}_t$.

$$\tilde{w}_t = \tilde{a}_t + \tilde{b}_t i = \frac{\varpi_t^L + \varpi_t^U + \left(1 - \rho_t^L\right) + \left(1 - \rho_t^U\right)}{4} + \sqrt{\frac{\left(\varpi_t^L - \bar{w}\right)^2 + \left(\varpi_t^U - \bar{w}\right)^2 + \left(\rho_t^L - \bar{w}\right)^2 + \left(\rho_t^U - \bar{w}\right)^2}{3}}\, i \tag{20}$$

where $\bar{w} = \frac{\varpi_t^L + \varpi_t^U + \rho_t^L + \rho_t^U}{4}, \forall t = 1, 2, \cdots, n.$

Step II-3: Calculate the weighted synthesis CN of each alternative.

We do the following sub-steps:

Step II−3.1: According to Eq. (3), compute the modulus of the attribute weight expressed by BCN in Eq. (20).

$$r_{\tilde{w}_t} = \sqrt{\tilde{a}_t^2 + \tilde{b}_t^2} \tag{21}$$

Then normalize the modulus of the attribute weight $r_{\tilde{w}_t}$ as

$$r_{\tilde{w}_t}^{'} = \frac{r_{\tilde{w}_t}}{\sum r_{\tilde{w}_t}}, \forall t = 1, 2, \cdots, n \tag{22}$$

Step II−3.2: Calculate the weighted synthesis CN $\eta_k = (\forall k = 1, 2, \cdots, m)$ of each scheme, shown as follows:

$$\eta_k = a_k + b_k i = \sum_{t=1}^{n} r'_{\tilde{w}_t} (a_{kt} + b_{kt} i) \quad (23)$$

Step II-4: Utilize Eq. (18) to construct the decision model η'_k based on the TCN.

Step II-5: Utilize Eq. (19) to calculate the first-order total PCN of the decision model obtained at Step II-4.

Step II-6: Calculate the value of the first-order total PCN obtained at Step II-5 when i^+ and i^- take various values and determine the corresponding ranking and the possible optimal scheme.

Step II-7: Utilize Eq. (12) to calculate the second-order total PCN of the decision model, then obtain the second-order ranking and optimal scheme.

Case III: Assume that the information about the attribute weights is completely unknown, then we need to determine the attribute weight first.

Because the attribute weight is unknown, the uncertainty of the attribute weight will cause the uncertainty of the scheme ranking. In general, if the difference in attribute values is smaller, it means that the attribute weight has less effect on the decision making. Conversely, if an attribute can make the attribute values of all schemes have great differences, it means that the attribute will play an important role in the decision making. Therefore, from the perspective of ranking or selecting the best scheme, no matter how important the attribute value itself is, the larger the deviation of the attribute values is, the greater the weight should be assigned, and the smaller the deviation is, the smaller the weight should be given. In particular, if there is no difference in the attribute values of all schemes, this attribute will have no effect on the decision making, and its weight can be zero Zhao (2005).

Based on the above principles, and considering that the attribute values have been converted from IVIFNs into the BCNs in Case I according to Eq. (16), the modulus of the BCN becomes a representative point of the original attribute value. The modulus of the BCN in Eq. (16) can be calculated by

$$\delta_{kt} = \sqrt{a_{kt}^2 + b_{kt}^2} \quad (24)$$

Calculate the variance of the modulus of each attribute a_t as follows:

$$D_t = \frac{\sum \left(\delta_{kt} - \bar{\delta}_{kt}\right)^2}{m - 1} \quad (25)$$

where $\bar{\delta}_{kt}$ is the average value of the weight moduli.

Then normalize the D_t value to obtain the attribute weight w_t:

$$w_t = \frac{D_t}{\sum D_t}, \forall t = 1, 2, \ldots, n \quad (26)$$

Now we get the most desirable alternative(s) according to the following steps:

Step III-1: Utilize Eqs. (13–16) to transform the interval-valued intuitionistic fuzzy decision matrix $D = [\alpha_{kt}]_{m\times n}$ into the binary CN decision matrix $R = [r_{kt}]_{m\times n} = [a_{kt} + b_{kt} i]_{m\times n}$.

Step III-2: Utilize Eqs. (24–26) to determine each attribute weight w_t.

Step III-3: Utilize Eq. (17) to calculate the weighted comprehensive CN η_k of each scheme S_k.

Step III-4: Utilize Eq. (18) to construct the decision model η'_k based on the TCN.

Step III-5: Based on Eq. (19), we can calculate the first-order total PCN $\partial^{\pm}\eta'_k$ of the decision model η'_k.

Step III-6: Calculate the various values of the first-order total PCN $\partial^{\pm}\eta'_k$ when i^+ and i^- take different values and determine the corresponding sorting and optimal scheme in the case of first-order uncertainties.

Step III-7: Utilize Eq. (12) to calculate the second-order total PCN $\partial^{2\pm}\eta'_k$ of the decision mode η'_k to obtain the second-order ranking and optimal scheme.

Step III-8: Based on the results of Step III-6 and Step III-7, give some suggestions for the decision making.

4 Illustrative example

In this section, three examples for IVIFMADM are used to demonstrate the applications and the effectiveness of the proposed method in three cases, that is, the attribute weights are known as real numbers, the attribute weights are known as IVIFNs, and the weights are completely unknown.

Example 4.1 Chen et al. (2012) An investment company conducts risk assessments for four alternative companies S_k $(\forall k = 1, 2, 3, 4)$: S_1 (automotive company), S_2 (food company), S_3 (IT company), S_4 (weaponry equipment company). According to the three evaluation attributes: A_1 (risk analysis), A_2 (growth analysis), A_3 (environmental impact analysis), the investment company chooses the best investment company from four companies. Assume that the weights of the attributes A_1, A_2, A_3 are crisp values, and the known attribute weighting information is given as follows: $w_1 = 0.35$, $w_2 = 0.25$, $w_3 = 0.4$ respectively.

After statistical processing, the evaluation information of each candidate company given by the decision maker can be expressed by the evaluation matrix containing IVIFNs as follows:

$$D = \begin{bmatrix} ([0.4, 0.5], [0.3, 0.4]) & ([0.4, 0.6], [0.2, 0.4]) & ([0.1, 0.3], [0.5, 0.6]) \\ ([0.6, 0.7], [0.2, 0.3]) & ([0.6, 0.7], [0.2, 0.3]) & ([0.4, 0.7], [0.1, 0.2]) \\ ([0.3, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.1, 0.3]) \\ ([0.7, 0.8], [0.1, 0.2]) & ([0.6, 0.7], [0.1, 0.3]) & ([0.3, 0.4], [0.1, 0.2]) \end{bmatrix}$$

The procedure of the proposed MADM method in Case I is shown as follows:

Step 1: Convert decision matrix D into the BCN matrix R by using Eqs. (13), (14) and (15).

$$R = \begin{bmatrix} 0.5500 + 0.0817i & 0.6000 + 0.1633i & 0.3250 + 0.2217i \\ 0.7000 + 0.2380i & 0.7000 + 0.2380i & 0.7000 + 0.2646i \\ 0.5500 + 0.1414i & 0.6000 + 0.1291i & 0.6750 + 0.2217i \\ 0.8000 + 0.3512i & 0.7250 + 0.2754i & 0.6000 + 0.1291i \end{bmatrix}$$

Step 2: The weighted comprehensive connection degree η_k of each scheme S_k $(k = 1, 2, 3, 4)$ is calculated by Eq. (17), and its value is as follows:
$\eta_1 = 0.4725 + 0.1581i$,
$\eta_2 = 0.7000 + 0.2487i$,
$\eta_3 = 0.6125 + 0.1705i$,
$\eta_4 = 0.7013 + 0.2434i$.
Step 3: The TCN of each scheme is obtained by using Eq. (18).
$\eta_1' = 0.4725 + 0.1581i + 0.3694j$,
$\eta_2' = 0.7000 + 0.2487i + 0.0513j$,
$\eta_3' = 0.6125 + 0.1705i + 0.2170j$,
$\eta_4' = 0.7013 + 0.2434i + 0.0554j$.
Step 4: The first-order total PCN of the decision CN for each scheme is obtained by utilizing Eq. (19).

$$\begin{aligned} \partial^{\pm}\eta_1 &= 0.7493 + 0.2997i^{+} + 0.2507i^{-} + 0.7003j \\ &= 0.0490 + 0.2997i^{+} + 0.2507i^{-}, \partial^{\pm}\eta_2 \\ &= 0.7379 + 0.8289i^{+} + 0.2621i^{-} + 0.1711j \\ &= 0.5667 + 0.8289i^{+} + 0.2621i^{-}, \partial^{\pm}\eta_3 \\ &= 0.7823 + 0.4399i^{+} + 0.2177i^{-} + 0.5601j \\ &= 0.2222 + 0.4399i^{+} + 0.2177i^{-}, \partial^{\pm}\eta_4 \\ &= 0.7423 + 0.8147i^{+} + 0.2577i^{-} + 0.1853j \\ &= 0.5571 + 0.8147i^{+} + 0.2577i^{-}. \end{aligned}$$

Step 5: The first-order total PCNs are calculated under uncertainty, and the possible optimal schemes and their rankings are discussed, shown as Table 1.
It can be seen from Table 1 that there exits $0.3046 > 0.2994 > 0.0045 > -0.2017$ when $i^{+} = 0, i^{-} = -1$ in the third column, so the order of the four schemes from superior to inferior is $S_2 \succ S_4 \succ S_3 \succ S_1$, which is consistent with the literature Chen et al. 2012. There exists $0.5667 > 0.5571 > 0.2222 > 0.0490$ in the fourth column, so the order of the four schemes from superior to inferior is still $S_2 \succ S_4 \succ S_3 \succ S_1$. Similarly, the order is also $S_2 \succ S_4 \succ S_3 \succ S_1$ in the fifth and sixth column. It seems that the scheme S_2 is the optimal solution, but the above is synchronized with i^{+} and i^{-} via the first-order total PCNs $\eta_1', \eta_2', \eta_3', \eta_4'$ of the four decision making CNs. If not, there may be different rankings. The reader can realize the order of the four alternatives when i^{+} and i^{-} are not synchronized. Not only can S_4 be the optimal solution but S_3 can also be the best solution. The only scheme that cannot be the optimal solution is S_1, but S_1 is not always the worst solution. For example, when column 5, row 2 (0.3487) in Table 1 and column 4, row 4 (0.2222) appear in the same scheme combination, scheme S_1 takes precedence over S_3.
Step 6: The second-order total PCNs of the four decision CNs are calculated, and the optimal scheme is determined according to the values of the second-order total PCNs. The maximum valued is the optimal scheme. The results are as follows:
$\partial^{2\pm}\eta_1 = -0.0221$,
$\partial^{2\pm}\eta_2 = 0.0760$,
$\partial^{2\pm}\eta_3 = -0.0800$,
$\partial^{2\pm}\eta_4 = 0.0585$,
Because $0.0760 > 0.0580 > -0.0221 > -0.0800$, the preference order of the four schemes is $S_2 \succ S_4 \succ S_1 \succ S_3$. S_2 is the optimal scheme, S_4 is the suboptimal scheme and they have the upward trend. The last one is S_3 and its development trend is declining.
Step 7: The conclusion obtained by combining Step 5 and Step 6 indicates that when the uncertainty of each intuitive attribute value of the four schemes under the first-order uncertainty environment is determined by synchronous change, the ranking of the four schemes is $S_2 \succ S_4 \succ S_3 \succ S_1$, and the optimal scheme is S_2.

Table 1 Values of the first-order total PCNs in the case of uncertainty when the attribute weights are real numbers

Schemes	$\partial^{\pm}\eta_k$	$i^{+}=0$, $i^{-}=-1$	$i^{+}=0$, $i^{-}=0$	$i^{+}=1$, $i^{-}=0$	$i^{+}=1$, $i^{-}=-1$
S_1	$0.0490 + 0.2997i^{+} + 0.2507i^{-}$	−0.2017	0.0490	0.3487	0.0980
S_2	$0.5667 + 0.8289i^{+} + 0.2621i^{-}$	0.3046	0.5667	1.3956	1.1335
S_3	$0.2222 + 0.4399i^{+} + 0.2177i^{-}$	0.0045	0.2222	0.6621	0.4444
S_4	$0.5571 + 0.8147i^{+} + 0.2577i^{-}$	0.2994	0.5571	1.3718	1.1141

In other uncertain cases, the optimal scheme may be either S_4 or S_3. In the second-order uncertainty environment (the uncertainty factor at this time has been theoretically eliminated), the order of the four schemes is $S_2 \succ S_4 \succ S_1 \succ S_3$. To this end, it is recommended that decision makers make final decisions based on the actual situation, supplemental new information, previous decision making experience, or some decision preferences.

Example 4.2 Wang and Chen (2017) Let S_1, S_2, S_3 and S_4 be four alternatives and let A_1, A_2, and A_3 be three attributes, these four possible alternatives are to be evaluated with the interval-valued intuitionistic fuzzy information based on the above three attributes, as listed in the following decision making matrix $D_{4\times 3}\,(d_{kt})$.

$$D_{4\times 3}\,(d_{kt}) = \begin{bmatrix} ([0.40, 0.50]\,, [0.30, 0.40]) & ([0.40, 0.60]\,, [0.20, 0.40]) & ([0.10, 0.30]\,, [0.50, 0.60]) \\ ([0.58, 0.70]\,, [0.10, 0.30]) & ([0.61, 0.70]\,, [0.20, 0.30]) & ([0.42, 0.70]\,, [0.10, 0.20]) \\ ([0.30, 0.60]\,, [0.30, 0.40]) & ([0.50, 0.60]\,, [0.30, 0.40]) & ([0.50, 0.60]\,, [0.10, 0.30]) \\ ([0.61, 0.80]\,, [0.10, 0.20]) & ([0.60, 0.70]\,, [0.10, 0.30]) & ([0.30, 0.40]\,, [0.10, 0.20]) \end{bmatrix}$$

Assume that the weights w_1, w_2 and w_3 of the attributes A_1, A_2 and A_3 are interval-valued intuitionistic fuzzy values, shown as follows:

$w_1 = ([0.10, 0.40]\,, [0.20, 0.55])$,
$w_2 = ([0.20, 0.50]\,, [0.15, 0.45])$,
$w_3 = ([0.25, 0.60]\,, [0.15, 0.38])$.

The decision making process for MADM proposed in Case II is shown as follows:

Step 1 Utilize Eqs. (13–16) to convert the decision making matrix $D_{4\times 3} = (d_{kt})$ into the BCN matrix $R_{4\times 3}\,(r_{kt}) = [a_{kt} + b_{kt} i]_{4\times 3}$.

$$R_{4\times 3}\,(r_{kt}) = \begin{bmatrix} 0.5500 + 0.0817i & 0.6000 + 0.1633i & 0.3250 + 0.2217i \\ 0.7200 + 0.2713i & 0.7025 + 0.2402i & 0.7050 + 0.2660i \\ 0.5500 + 0.1414i & 0.6000 + 0.1291i & 0.6750 + 0.2217i \\ 0.7775 + 0.3322i & 0.7250 + 0.2754i & 0.6000 + 0.1291i \end{bmatrix}$$

Step 2 Utilize Eq. (20) to convert the interval-valued intuitionistic fuzzy attribute weight w_1, w_2 and w_3 into the corresponding BCN weight $\tilde{w_1}$, $\tilde{w_2}$, $\tilde{w_3}$, respectively.

$\tilde{w_1} = 0.4375 + 0.2016i$,
$\tilde{w_2} = 0.5250 + 0.1756i$,
$\tilde{w_3} = 0.5800 + 0.1943i$.

Step 3 Calculate the weighted synthesis CN η_k $(k = 1, 2, 3, 4)$ for each alternative. We do the following sub-steps.

Step 3.1 Compute the modulus of the attribute weight expressed by BCN in the previous Step 2 according to Eqs. (21), (22), their corresponding values are $r'_{\tilde{w}_1} = 0.2925$, $r'_{\tilde{w}_2} = 0.3361$, $r'_{\tilde{w}_3} = 0.3714$.

Step 3.2 Utilize Eq. (23) to calculate the weighted synthesis CN η_k $(k = 1, 2, 3, 4)$ of each scheme, shown as follows:

$\eta_1 = 0.4832 + 0.1611i$,
$\eta_2 = 0.7085 + 0.2589i$,
$\eta_3 = 0.6132 + 0.1671i$,
$\eta_4 = 0.6939 + 0.2377i$.

Step 4 Utilize Eq. (18) to construct the decision model based on the TCN for each scheme.

$\eta_1 = 0.4832 + 0.1611i + 0.3556j$,
$\eta_2 = 0.7085 + 0.2589i + 0.0326j$,
$\eta_3 = 0.6132 + 0.1671i + 0.2197j$,
$\eta_4 = 0.6939 + 0.2377i + 0.0684j$.

Step 5 Calculate the first-order total PCN $\eta_k^{\pm}$ $(k = 1, 2, 3, 4)$ of the decision model η_k $(k = 1, 2, 3, 4)$ obtained in the Step 4 according to Eq. (19).

$$\begin{aligned} \partial^{\pm}\eta_1 &= 0.7500 + 0.3118i^{+} + 0.2500i^{-} + 0.6882j \\ &= 0.0617 + 0.3118i^{+} + 0.2500i^{-},\ \partial^{\pm}\eta_2 \\ &= 0.7324 + 0.8882i^{+} + 0.2676i^{-} + 0.1118j \\ &= 0.6206 + 0.8882i^{+} + 0.2676i^{-},\ \partial^{\pm}\eta_3 \\ &= 0.7859 + 0.4321i^{+} + 0.2141i^{-} + 0.5679j \\ &= 0.2179 + 0.4321i^{+} + 0.2141i^{-},\ \partial^{\pm}\eta_4 \\ &= 0.7449 + 0.7765i^{+} + 0.2551i^{-} + 0.2235j \\ &= 0.5214 + 0.7765i^{+} + 0.2551i^{-}, \end{aligned}$$

Step 6 Calculate the first-order total PCNs of the four decision models under uncertainty, shown as Table 2 and discuss the possible optimal schemes and the preference order.

From Table 2, it can been found that when $i^{+} = 0, i^{-} = -1$ or $i^{+} = 0, i^{-} = 0$, or $i^{+} = 1, i^{-} = 0$, or $i^{+} = 0, i^{-} = -1$, the preference order of the four alternatives S_1, S_2, S_3, and S_4 is $S_2 \succ S_4 \succ S_3 \succ S_1$, which coincides with the ones shown in Wang and Chen (2017).

Step 7 Calculate the second-order total PCN $\eta_k^{2\pm}$ $(k = 1, 2, 3, 4)$ of the decision model obtained in Step 5, then obtain the second-order ranking according to Eq. (12).

$\eta_1^{2\pm} = -0.0271$,

Table 2 Values of the first-order total PCNs when the attribute weights are IVIFNs

Schemes	$\partial^{\pm}\eta_k$	$i^+=0$ $i^-=-1$	$i^+=0$ $i^-=0$	$i^+=1$ $i^-=0$	$i^+=1$ $i^-=-1$
S_1	$0.0617+0.3118i^+ + 0.2500i^-$	−0.1883	0.0617	0.3735	0.1235
S_2	$0.6206+0.8882i^+ + 0.2676i^-$	0.3530	0.6206	1.5089	1.2413
S_3	$0.2179+0.4321i^+ + 0.2141i^-$	0.0038	0.2179	0.6500	0.4358
S_4	$0.5214+0.7765i^+ + 0.2551i^-$	0.2663	0.5214	1.2979	1.0428

$\eta_2^{2\pm}=0.1573$,
$\eta_3^{2\pm}=-0.0809$,
$\eta_4^{2\pm}=0.0227$.

Because $\eta_2^{2\pm} > \eta_4^{2\pm} > \eta_1^{2\pm} > \eta_3^{2\pm}$ the preference order based on the development trend of each scheme is $S_2 \succ S_4 \succ S_1 \succ S_3$.

Step 8 Synthesize the results of Step 6 and Step 7, and then give decision making suggestions.

The evaluated schemes are developmental and growing, and they need to be comprehensively evaluated. No matter from the order of the status or the development trend for each scheme, S_2 is the optimal one. Therefore, it shows that its performance and development potential are all in the best status. S_2 should be regarded as the optimal scheme.

Example 4.3 Garg (2016) Let S_1, S_2, S_3, S_4 and S_5 be five alternatives which is to be evaluated by the decision maker according to the six different attributes A_1, A_2, A_3, A_4, A_5 and A_6. The decision matrix $D_{5\times6}\,(d_{kt})$ information related to these alternatives is shown in Tables 3 and 4. The information about attribute weights is completely unknown.

Step 1: Transform the interval-valued intuitionistic fuzzy decision matrix $D=[d_{kt}]_{5\times6}$ into the BCN decision matrix $R=[a_{kt}+b_{kt}i]_{5\times6}$ according to Eqs. (13–16) as

According to Eq. (17), we get the weighted comprehensive CN of each scheme as the following
$\eta_1=0.5357+0.2076i$,
$\eta_2=0.6216+0.2148i$,
$\eta_3=0.6309+0.1713i$,
$\eta_4=0.7023+0.2466i$,
$\eta_5=0.6352+0.1967i$.

Step 4: Construct the decision model $\eta_k^{'}$ based on the TCN.

According to Eq. (18), we can obtain the decision model $\eta_k^{'}$ based on the TCN as the following
$\eta_1=0.5357+0.2076i+0.2567j$,
$\eta_2=0.6216+0.2148i+0.1636j$,
$\eta_3=0.6309+0.1713i+0.1977j$,
$\eta_4=0.7023+0.2466i+0.0511j$,
$\eta_5=0.6352+0.1967i+0.1681j$.

Step 5: Calculate the first-order total PCN $\partial^{\pm}\eta_k^{'}$ of the decision model $\eta_k^{'}$.

We can get the first-order total PCN $\partial^{\pm}\eta_k^{'}$ of each decision model $\eta_k^{'}$ by utilizing Eq. (19), shown as follows:

$$
\begin{aligned}
\partial^{\pm}\eta_1 &= 0.7207+0.4471i^+ + 0.2793i^- + 0.5529j\\
&= 0.1678+0.4471i^+ + 0.2793i^-,\ \partial^{\pm}\eta_2\\
&= 0.7432+0.5677i^+ + 0.2568i^- + 0.4323j\\
&= 0.3108+0.5677i^+ + 0.2568i^-,\ \partial^{\pm}\eta_3
\end{aligned}
$$

$$
R=\begin{bmatrix}
0.4000+0.1291i & 0.7000+0.2380i & 0.5750+0.1258i & 0.8000+0.3512i & 0.3250+0.2217i & 0.6750+0.2217i\\
0.7000+0.2380i & 0.6750+0.2217i & 0.7000+0.2380i & 0.7500+0.2944i & 0.4000+0.1291i & 0.7000+0.2646i\\
0.5500+0.0817i & 0.8000+0.3512i & 0.6000+0.1291i & 0.7250+0.2754i & 0.5500+0.0817i & 0.6000+0.1633i\\
0.7000+0.2380i & 0.6750+0.2212i & 0.8000+0.3512i & 0.6000+0.1291i & 0.6750+0.2217i & 0.8000+0.3512i\\
0.5750+0.1258i & 0.4750+0.0957i & 0.7250+0.2754i & 0.7750+0.3304i & 0.7000+0.2380i & 0.6250+0.1708i
\end{bmatrix}
$$

Step 2: Determine the weight w_t with respect to each attribute.

We utilize Eqs. (24–26) to get the weight vector corresponding to each attribute as follows:
$w_1=0.1872$, $w_2=0.2012$, $w_3=0.1430$, $w_4=0.1053$, $w_5=0.2622$, $w_6=0.1011$.

Step 3: Calculate the weighted comprehensive CN η_k of each scheme S_k.

$$
\begin{aligned}
&= 0.7864+0.4642i^+ + 0.2136i^- + 0.5358j\\
&= 0.2507+0.4642i^+ + 0.2136i^-,\ \partial^{\pm}\eta_4\\
&= 0.7401+0.8284i^+ + 0.2599i^- + 0.1716j\\
&= 0.5685+0.8284i^+ + 0.2599i^-,\ \partial^{\pm}\eta_5\\
&= 0.7636+0.5392i^+ + 0.2364i^- + 0.4608j\\
&= 0.3027+0.5392i^+ + 0.2364i^-,
\end{aligned}
$$

Table 3 Decision matrix $D_{5\times 6}\left(d_{kt}\right)$

	A_1	A_2	A_3
S_1	$([0.2, 0.3], [0.4, 0.5])$	$([0.6, 0.7], [0.2, 0.3])$	$([0.4, 0.5], [0.2, 0.4])$
S_2	$([0.6, 0.7], [0.2, 0.3])$	$([0.5, 0.6], [0.1, 0.3])$	$([0.6, 0.7], [0.2, 0.3])$
S_3	$([0.4, 0.5], [0.3, 0.4])$	$([0.7, 0.8], [0.1, 0.2])$	$([0.5, 0.6], [0.3, 0.4])$
S_4	$([0.6, 0.7], [0.2, 0.3])$	$([0.5, 0.6], [0.1, 0.3])$	$([0.7, 0.8], [0.1, 0.2])$
S_5	$([0.5, 0.6], [0.3, 0.5])$	$([0.3, 0.4], [0.3, 0.5])$	$([0.6, 0.7], [0.1, 0.3])$

Table 4 Decision matrix $D_{5\times 6}\left(d_{kt}\right)$

	A_4	A_5	A_6
S_1	$([0.7, 0.8], [0.1, 0.2])$	$([0.1, 0.3], [0.5, 0.6])$	$([0.5, 0.7], [0.2, 0.3])$
S_2	$([0.6, 0.7], [0.1, 0.2])$	$([0.3, 0.4], [0.5, 0.6])$	$([0.4, 0.7], [0.1, 0.2])$
S_3	$([0.6, 0.7], [0.1, 0.3])$	$([0.4, 0.5], [0.3, 0.4])$	$([0.3, 0.5], [0.1, 0.3])$
S_4	$([0.3, 0.4], [0.1, 0.2])$	$([0.5, 0.6], [0.1, 0.3])$	$([0.7, 0.8], [0.1, 0.2])$
S_5	$([0.6, 0.8], [0.1, 0.2])$	$([0.6, 0.7], [0.2, 0.3])$	$([0.5, 0.6], [0.2, 0.4])$

Table 5 Values of the first-order total PCNs when the attribute weights are IVIFNs

Schemes	$\partial^{\pm}\eta_k$	$i^+ = 0$, $i^- = -1$	$i^+ = 0$, $i^- = 0$	$i^+ = 1$, $i^- = 0$	$i^+ = 1$, $i^- = -1$
S_1	$0.1678 + 0.4471i^+ + 0.2793i^-$	−0.1115	0.1678	0.6149	0.3356
S_2	$0.3108 + 0.5677i^+ + 0.2568i^-$	0.0540	0.3108	0.8785	0.6217
S_3	$0.2507 + 0.4642i^+ + 0.2136i^-$	0.0371	0.2507	0.7149	0.5014
S_4	$0.5685 + 0.8284i^+ + 0.2599i^-$	0.3086	0.5685	1.3969	1.1370
S_5	$0.3027 + 0.5392i^+ + 0.2364i^-$	0.0663	0.3027	0.8419	0.6055

Step 6: Calculate various values of the first-order total PCN $\partial^{\pm}\eta_k'$ when i^+ and i^- take different values and discuss the possible optimal schemes and their rankings. The values of the first-order total PCN $\partial^{\pm}\eta_k'$ $(k = 1, 2, 3, 4, 5)$ in Step 5 when i^+ and i^- take different values are shown in Table 5 and discuss various sorting and optimal scheme in the case of first-order uncertainty.

From Table 5, it can be found that there exits the preference order of the five schemes is $S_4 \succ S_5 \succ S_2 \succ S_3 \succ S_1$ when $i^+ = 0, i^- = -1$ or $i^+ = 0, i^- = 0$, which is consistent with the result in Zhao 2005. But when $i^+ = 1, i^- = 0$ or $i^+ = 1, i^- = -1$, the preference order is $S_4 \succ S_2 \succ S_5 \succ S_3 \succ S_1$. In the environment affected by a variety of uncertainties, we need to examine different results. The above calculation shows only two different results, but these two rankings indicate that S_4 is the best scheme and S_1 is the worst. Further, we calculate the second-order PCN of the decision model.

Step 7 Utilize Eq. (12) to calculate the second-order total PCN $\partial^{2\pm}\eta_k'$ of the decision model η_k' to obtain the second-order ranking and optimal scheme.

According to Eq. (12), the second-order total PCN $\partial^{2\pm}\eta_k'$ can be calculated as follows:

$\partial^{2\pm}\eta_1' = -0.0472,$
$\partial^{2\pm}\eta_2' = -0.0604,$
$\partial^{2\pm}\eta_3' = -0.0862,$
$\partial^{2\pm}\eta_4' = 0.0742,$
$\partial^{2\pm}\eta_5' = -0.0748.$

It follows that the second-order PCN in the scheme S_4 is positive, and the development trend is positive. The second-order PCNs in the other schemes are negative, the development trend is not optimistic. Therefore, it is finally determined that S_4 is the optimal solution.

5 Discussion

In this paper, we investigated IVIFMADM problems with attribute weight represented by real numbers, IVIFNs or with incomplete attribute weight information. New methods were developed to solve such problems. The primary contributions of this research are outlined as follows:

(1) It is well known that given an interval number $X = \left[x^L, x^U\right]$, $\left(x^L \le x^U\right)$, $X \in R$, there are an infinite number of x values satisfying the condition $x^L \le x \le x^U$. Constructing the IVIFN $\alpha_x = \left\langle\left[\mu_x^L, \mu_x^U\right], \left[\nu_x^L, \nu_x^U\right]\right\rangle$ with such an interval number x naturally includes infinite possible values of x. Therefore, theoretically, there are infinite solutions for MADM problems described by IVIFNs. A specific algorithm is designed for a specific IVIFMADM problem to give a unique and definite rank-

ing of the decision making schemes and to determine the optimal scheme, which is only responsible for the specific algorithm and does not guarantee that the scheme will be optimal in the actual decision making environment. In fact, only when the known condition of a problem is completely determined, then there is a completely certain answer; when the known condition of a problem is uncertain, the corresponding answer must be uncertain, which requires uncertainty analysis to give an acceptable conclusion. Therefore, for the MDAM problem described by IVIFNs, it is logical to have a link of uncertainty analysis. On the other hand, the interval number has the certainty of the upper and lower bounds. It is meaningful to consider the uncertainty of interval numbers and to use the certainty of interval numbers to make decisions in a specific intuitionistic fuzzy MADM problem. In this paper, the MADM problem described by IVIFNs is analyzed at two levels. First, we convert the attribute values represented by IVIFNs into TCNs; then, we the uncertainty of the total PCN at the first- order level and the deterministic calculation at the second-order level. The first level is the level of the first-order total PCN, and the second level is the level of the second-order total PCN. To distinguish PCN level from the commonly used level concept, this paper uses order to represent level, and the title of the paper is taken as interval-valued intuitionistic fuzzy multi-attribute second-order decision making based on PCNs.

(2) Another innovation of this paper is that the IVIFMADM model is regarded as a model with an internal positive–negative contradiction movement mechanism. This paper holds that: First, intuition is a way of thinking in motion, and second, the understanding of fuzziness in the human brain is a type of thinking movement Jiang and Zhao (2019). Therefore, the IVIFMADM is a choice within a specific thinking movement. Based on these considerations, after transforming IVIFNs into CNs, it is natural to introduce the concept of PCNs and to quantitatively study the uncertain motion hidden in the interval-valued intuitionistic fuzzy MADM via PCNs. Additionally, it is natural and reasonable to formally express the partial positive motion depicted by the partial positive CNs and the partial negative motion depicted by the partial negative CNs and their synthesis.

(3) Interval-valued intuitionistic fuzzy MADM is an uncertain decision making tool. Due to the diversity of uncertainties, the interval-valued intuitionistic fuzzy MADM problems have many decision making models and algorithms. In this paper, a novel decision making model and algorithm are described. The practical examples in this paper show that the model has better flexibility and applicability.

6 Conclusion

The PCN is an adjoint function of the CN. Its calculation process and results describe the mutual connection, mutual restriction and mutually generated contradictory movement of all connection components in the CN at the micro-level, which has rich systematic information. PCN algorithm is a new intelligent algorithm. First, from the perspective of information utilization, the PCN algorithm effectively mines the dynamic information of the connection component in the CN, which reflects the essence of the research object described by the CN. Secondly, from the perspective of artificial intelligence technology innovation, the research on clustering, pattern recognition, system comprehensive evaluation decision making, and risk prevention and control based on the PCN, and privacy protection research in social networks also belong to the category of intelligent technology in a certain sense. Therefore, the PCN algorithm is a new intelligent algorithm, which needs to be deeply and systematically studied. At present, the PCN is still weak in theoretical basis and practical application. In order to further develop its theory and method, the problems that need to be further studied mainly focus on the following aspects.

(1) Currently, there are various expressions of PCNs, and it is necessary to further compare and study these expressions to standardize and unify as much as possible or clarify the corresponding scope of application.

(2) According to the value of the PCN, the development trend of things can be roughly judge. However, due to the uncertainty of the value of uncertainty coefficient i in the PCN, the obtained trend judgment conclusion will still have a certain degree of uncertainty. How to reduce the uncertainty of the trend judgment conclusion and enhance the credibility of the trend judgment conclusion usually requires a comprehensive analysis. These analyses include the structural analysis of each connection component in the original CN, the cause analysis of each connection component, as well as analysis of the environment in which the raw data resides, etc.

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Data availability Enquires about data availability should be directed to the authors.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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